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# UNITED STATES
# EXPERIMENTAL MODEL BASIN

### NAVY YARD, WASHINGTON, D.C.

## A STUDY OF SHIP HULL VIBRATION
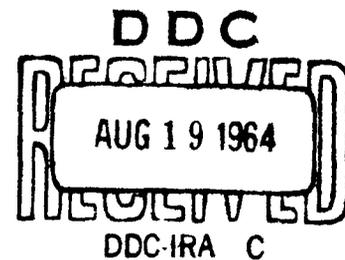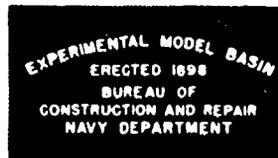
BY R. T. McGOLDRICK

FEBRUARY 1935                    REPORT NO. 395

# A STUDY OF SHIP HULL VIBRATION

## by R. T. McGoldrick

U. S. Experimental Model Basin

Navy Yard, Washington, D. C.

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# FIGURES

## 1. Summary

This report deals with the various types of vibration of the complete hull structure which may be set up by periodic forces which synchronize with one of the natural frequencies. It includes a review of the investigations made in the field of hull vibration from the original studies of O. Schlick in 1884 up to date. It also gives an account of model experiments and full scale measurements conducted recently at the U. S. Experimental Model Basin. Particular attention is given to the methods of J. L. Taylor, E. Schadlofsky, and F. M. Lewis which are considered the most practical of the theoretical methods. The limitations of these methods are pointed out and the relation between the vibration problem and the strength problem is emphasized. An appraisal is given of the method of F. M. Lewis for estimating the effect of the surrounding water on the natural frequency. The conclusion is drawn that for naval vessels the most practical procedure is to estimate the natural frequencies from measurements previously made on ships of the same class by selecting the appropriate value of the Schlick constant. The graphical methods are recommended only for cases of radically new design where empirical constants are not available.

## 2. Introduction

Ship vibration has been receiving increasing attention since the pioneer work of O. Schlick (1)[*] in 1884, and at present many data are available both on the practical and theoretical sides of this problem. The calculation of the two-noded vertical flexural frequency of a ship is an interesting problem and has been given by far the most attention by theoretical investigators. As for other possible types of vibration such as horizontal, torsional, and longitudinal, while theoretical solutions have been proposed the data required for the use of these methods are usually not available and can be obtained only by laborious computations. As will be shown, even in the case of the vertical, two-noded vibration there are still several doubtful correction factors of considerable magnitude which must be applied after making the theoretical computation, and it would seem that much of the effort spent on the academic problem of the natural frequencies of non-uniform bars might more profitably have been spent in collecting data on actual frequencies from which practical engineers could forecast the critical frequencies of proposed designs.

One justification for the consideration given the vertical two-noded frequency is its close relation to the bending strength of the vessel. Both the strength of the ship under static or dynamic load and its two-noded vertical frequency depend upon the moment of inertia of its section about a horizontal axis. While for the same type of section and same displacement the natural frequency will vary with the distribution of the load, the general principle holds that the higher the natural frequency the greater the bending strength of the ship.

[*] Numbers in parentheses correspond to references in the bibliography at the end of the report.

The measurement of natural frequencies has proved to be of practical importance in the design of bridges and should also be of value in ship design. The action of the surrounding water, however, renders the ship problem more complicated. Purely from the strength point of view a ship should be designed so as to have the highest attainable vertical two-noded frequency under a fixed condition of loading, in accordance with the general principle of mechanics that the natural frequency varies as $\sqrt{\dfrac{\text{rigidity}}{\text{mass}}}$ .

### 3. Calculation of the Two-noded Vertical Frequency

#### a. Derivation of the Theoretical Frequency

In the calculation of the flexural natural frequencies the ship is considered as a bar of non-uniform cross-section with free ends. The problem of the free-free bar of uniform cross-section was solved by Lord Rayleigh (2) and the natural frequency of the two-noded flexural vibration can be expressed by the simple formula:

In this formula
$$n = \frac{9}{8}\pi\sqrt{\frac{EI}{WL^3}} \quad \dots \dots \dots \dots \dots \dots \dots \dots \quad (1)$$

$E$ = modulus of elasticity in $lb/in^2$

$n$ = frequency in vibrations per second

$I$ = moment of inertia of section in $ft^2$ $in^2$ units

$L$ = length of bar in feet

$W$ = mass of bar in slugs (weight in $lb/32.2$)

For ships, frequencies are usually expressed in vibrations per minute and the displacement is given in tons. Also the modulus of structural steel can be given the common value $30 \times 10^6$ lb. per sq. in. without serious error. Using these quantities the free-free bar formula becomes

$$N = 60 \times \frac{9\pi}{8}\sqrt{\frac{30 \times 10^6 \times I}{D \times \frac{2240}{32.2} \times L^3}} \quad = 1.39 \times 10^5\sqrt{\frac{I}{DL^3}} \quad \dots \dots \dots \dots \quad (2)$$

where $N$ is expressed in vibrations per minute and $D$ is the displacement or weight in tons.

In 1894 O. Schlick (1) proposed an empirical formula for the two noded flexural frequency of a ship similar to the above formula:

$$N = c\sqrt{\frac{I}{DL^3}} \quad \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \quad (3)$$

where c is an experimental constant called "Schlick's constant" and varies from $1.28 \times 10^5$ to $1.57 \times 10^5$ according to the type of vessel. In this formula N is vibrations per minute, I is the moment of inertia of the midship section in $ft.^2$ $in.^2$, D is the displacement in tons, and L is the length in feet.

This formula failed to check measured values in numerous cases and it was recognized that a more accurate calculation was required to take into account the

non-uniformity of weight distribution and the variation in the ship section.

A theoretical solution for the flexural natural frequencies of a free-free non-uniform bar may be obtained by solving the differential equation

$$\frac{d^2}{dx^2}\left(EI\,\frac{d^2y}{dx^2}\right) + m\,\frac{d^2y}{dt^2} = 0 \quad . . . . . . . . . . . . . . (4)$$

Here x represents a distance along the length of the bar, y represents the vertical displacement of any point from its normal position, E is the modulus of elasticity, I is the moment of inertia of the cross section, t represents time, and m is the mass per unit length. This equation is base on the flexure of slender beams and gives the relation between the second derivative of the bending moment with respect to length and the dynamic load (mass times acceleration) at any instant during vibration. If the amplitudes are small in proportion to the length the assumption may be made that every point of the bar describes a linear simple harmonic motion in the y direction. If $w = 2\pi$ times the natural frequency, and y is the amplitude at any point, the acceleration at the instant of maximum flexure is $w^2y$. Considering the bar at this instant equation (4) becomes

$$\frac{d^2}{dx^2}\left(EI\,\frac{d^2y}{dx^2}\right) - mw^2y = 0 \quad . . . . . . . . . . . . . . (5)$$

It is possible to deduce the frequency from equation (5) in certain cases where m and I are known functions of x. In the case of the ship, however, these functions cannot be expressed in simple analytical form.

Lord Rayleigh (2) showed that if in the case of a non-uniform bar the variation of amplitude along the length were assumed to be the same as in the case of a uniform bar the error in the calculated frequency would be slight. On this basis an approximate solution for the frequency of a ship can be developed by assuming that at the instant of maximum flexure during vibration the relative deflections at points along the length are proportional to the amplitudes in a uniform bar. Thus the case is analogous to that of a static beam except that the deflections are due to dynamic loads which at any point are equal to the product of the mass and the acceleration at that point. If the amplitudes are small any point can be considered to be executing a linear simple harmonic motion and at maximum deflection the acceleration is $4\pi^2\,n^2y$, where n is the natural frequency and y is the amplitude.

The assumption of an amplitude profile makes it possible to compute the dynamic load ($4\pi^2\,n^2\,my$) as a function of length in relative terms which are proportional to the unknown frequency. The actual value of the deflections may then be deduced from the elastic properties of the hull just as is done in the strength calculations of ships. From the absolute value of deflections due to an assumed dynamic load the frequency may then be determined.

A solution can also be obtained for the frequency with the same assumption of an amplitude profile by equating the potential energy at the position of maximum

deflection to the kinetic energy in passing through the position of maximum velocity.

In the bibliography will be found numerous articles of interest in the historical development of these methods. This report is confined to what seem to be the most practical methods at the present time, namely the graphical methods of J. L. Taylor (5) and E. Schadlofsky (6) and the tabular method of F. M. Lewis (7). It should be noted that these calculations give only the theoretical frequency $(N_{th})$ namely the frequency which the ship would have if there were no water effect and if the deflections were due only to simple flexure as in the case of slender beams.

Taylor's method may be summarized briefly as follows:- If the amplitude at the forward perpendicular is assumed to be such as to make the acceleration at maximum deflection equal to the acceleration of gravity, the dynamic load at this point will be equal to the weight and at any other point will be obtained by multiplying the weight by the corresponding amplitude, the assumption also being made that the amplitudes vary as in a uniform bar whose end amplitude is unity. Having the dynamic load curve the next step is to find the resulting deflection by the usual graphical method applicable to beams which involves four integrations. If the initially assumed relative amplitudes were correct the deflection curve thus obtained would correspond to the initial curve except for the scale factor of integration. By reading the absolute value of the amplitude at the forward perpendicular which would make the acceleration equal to "g" the theoretical frequency is found by solving for n in the equation

$$4\pi^2 n^2 y_{v.p.} = g \quad \ldots \ldots \ldots \ldots \ldots \ldots \ldots (6)$$

where $y_{v.p.}$ is the computed amplitude at the forward perpendicular.

The actual procedure is as follows: the curve of relative amplitudes for the free-free uniform bar is plotted (the end value being made equal to unity). These values quoted from Taylor's paper, are as follows:

| Station | 0 | ½ | 1 | 1½ | 2 | 2½ | 3 | 3½ | 4 | 4½ | 5 |
|---------|-----|-----|------|------|------|-------|-------|-------|-------|-------|-------|
|         | 10  | 9½  | 9    | 8½   | 8    | 7½    | 7     | 6½    | 6     | 5½    |       |
| Rel.Amp. | 1.0 | .768 | .537 | .313 | .097 | -.099 | -.272 | -.414 | -.521 | -.586 | -.608 |

The weight curve is then multiplied by the corresponding amplitude thus giving the dynamic load curve which will be of negative sign between the nodes if the amplitudes are assumed positive at the ends. Fig. (1) shows the graphical computation for the U.S.S. HAMILTON by Taylor's method which may be used as an illustration. Upon integrating the dynamic load curve by means of the integraph it will in general be found that the shear curve does not close as required by dynamics. Taylor recommends joining the two ends of the shear curve and reading off the values with

respect to this inclined base line when the residual shear is small. The replotted shear curve is then integrated giving the moment curve which probably will not close. As dynamics requires the moment to be zero at the forward perpendicular Taylor joins the ends of the moment curve, reading off the moment values from the inclined base. Taylor points out, however, that the strictly correct method of fulfilling the end conditions is to revise the assumed amplitude curve by a parallel shift and a rotation of its base and thus obtain a new set of amplitude factors which when multiplied by the weight curve will give a new dynamic load curve.

The precise method of making these base corrections has been clearly shown by E. Schadlofsky (6). The first dynamic load curve is integrated twice giving a shear and a moment curve neither of which closes. The residual values of shear and moment at the forward perpendicular are designated as $RS_{1_{v.p.}}$ and $RM_{1_{v.p.}}$ respectively. In order to make the base corrections by Schadlofsky's method the following procedure must be followed. First must be determined the quantity $J_G = \Sigma\, x^2 \Delta W$ where x is the distance of an element of weight W from a plane passing through the ship's center of gravity and perpendicular to the x axis. This quantity $J_G$ is numerically equal to the moment of inertia of the area under the weight curve about a vertical line through the center of area. The distance from the center of gravity to the forward perpendicular is designated as $l_2$. The correction of the base of the assumed amplitude curve is made first by a parallel shift designated by $y_s$ and second by a rotation of the shifted base about the point where it intersects the vertical line through the center of gravity. The amount by which the rotation raises or lowers the base at the forward perpendicular is designated by $y_{o_{v.p.}}$. The values of these terms are

$$y_s = \frac{RS_{1_{v.p.}}}{\text{weight of ship}} \quad \ldots \ldots \ldots \ldots \ldots \ldots \quad (7)$$

and

$$y_{o_{v.p.}} = \frac{-l_2\left[RM_{1_{v.p.}} - RS_{1_{v.p.}} \times l_2\right]}{J_G} \quad \ldots \ldots \ldots \quad (8)$$

both $y_s$ and $y_{o_{v.p.}}$ being dimensionless.

[Note: Formula (8) differs from the formula for $y_{o_{v.p.}}$ given in Eq. (7) of Schadlofsky's paper. A recheck of the derivation showed that the terms $(y_{1_{v.p.}} - y_s)$ should be omitted.]

The amplitude values measured from the shifted and rotated base are then divided by the ordinate at the forward perpendicular. Thus is obtained a new amplitude curve which will satisfy both end conditions and at the same time has unity value at the forward perpendicular. The previously outlined method of obtaining the dynamic load, shear, and moment curves is then repeated using the new set of amplitude factors. Schadlofsky points out that the assumption of any set of amplitude factors that fails to satisfy the end conditions will give too low a value for the theoretical frequency.

Having obtained the moment curve by either Taylor's or Schadlofsky's method the remaining steps are the same for both methods. First the moment values are read off for twenty sections and divided by the corresponding values of moment of inertia of the section and the curve of M/I is plotted. A double integration of this M/I curve gives the curve Ey from which the deflections produced by the assumed dynamic load may be deduced.

In carrying out these steps several points should be noted. After each integral is drawn by the integraph its base must be located which is equivalent to determining the constant of integration. In the case of the shear curve the base is determined by the condition that the vertical shear must be zero at the ends since there are no external forces. For the same reason the moment must be zero at the ends, and the M/I curve which is a plotted curve is therefore zero at both ends. The curve obtained by integration of the M/I curve is usually referred to as the slope curve since its ordinates represent the slope of the final deflection or Ey curve. The base for the slope curve is usually selected so as to intersect it about amidships. After integrating the slope curve with respect to this base the ends of the final Ey curve are jointed and the connecting line is used as a basis of measurement. This eliminates any error due to the incorrect selection of the base of the slope curve as the effect of integrating a curve with respect to a wrong base is to tilt the integral curve but not to change values measured vertically from a line joining the two ends. The proper value of Ey at the forward perpendicular is then obtained as follows: Denote by $(Ey)_{max}$ the maximum value of the Ey curve measured vertically from the line joining the ends, and let $y_{v.p.}$ be the assumed amplitude at the forward perpendicular after making the base corrections. Let $y_{max}$ be the maximum amplitude measured from the line joining the ends of this corrected amplitude curve. Then:

$$Ey_{v.p.} = (Ey)_{max} \times \frac{y_{v.p.}}{y_{max}}$$

The value of Ey at the forward perpendicular is then divided by E which gives $y_{v.p.}$ (the amplitude required to make the acceleration at the forward perpendicular equal to the acceleration of gravity). The frequency is found by formula (6) previously given. The value of E to be used should be the average test piece value of the steel used in the hull. The practice of using a reduced value of E has been superseded by the corrections made to the theoretical frequency for the effect of elastic behavior.

There is no way of checking the theoretical frequency of a ship by actual measurement because of the water effect, but Schadlofsky has checked it for several dynamic models of ships, vibrating them out of the water.

For comparison graphical computations are shown for the U.S.S. HAMILTON by both Taylor's simplified method and the method of Schadlofsky (see Figs. 1 and 2).

In the Taylor computation $N_{th}$ was deduced as follows: The midship value of the Ey curve measured vertically from the line joining the ends is 14.03". The

TWO-NODED VIBRATION CALCULATION
BY TAYLOR'S SIMPLIFIED METHOD

LONGITUDINAL SCALE-I INCH=7.75 FT.
A-DYNAMIC LOAD-I INCH=.667 TONS/FT.
B-SHEAR-I INCH=36.2 TONS
C-BENDING MOMENT-I INCH=1400 TON FT.
D-M/I-I INCH=.0560 TON/INCH² FT.
E-SLOPE-I INCH=3.47 TON/INCH²
F-EY-I INCH=228.7 TON FT./INCH²

NOTE -
DUE TO THE USE OF THE INTEGRAPH
EACH INTEGRAL CURVE IS DISPLACED
TO THE LEFT OF THE CURVE INTEGRATED

U.S.S.

FRAME NO.  177

Fig. 1.  Graphical Computation

**2**

U.S.S. HAMILTON

A

D

C

B

B

I SQ. IN.

88 1/2

O

ical Computation for U.S.S. HAMILTON by Taylor's Method.

**1**



U.S.S. HAMILTON

NOTE -
DUE TO THE USE OF THE INTEGRAPH
EACH INTEGRAL CURVE IS DISPLACED
TO THE LEFT OF THE CURVE INTEGRATED

F

B

A

TWO-NODED VIBRATION CALCULATION
BY SCHADLOFSKY'S METHOD-BASE
CORRECTIONS PREVIOUSLY MADE

LONGITUDINAL SCALE - I INCH = 7.75 FT.
A-DYNAMIC LOAD - I INCH = .667 TONS/FT
B-SHEAR - I INCH = 20.7 TONS
C-BENDING MOMENT- I INCH = 881 TON FT
D-M/I - I INCH = .0392 TON/INCH$^2$FT
E-SLOPE- I INCH = 3.23 TON/INCH$^2$
F-EY- I INCH = 200.2 TON FT/INCH$^2$

E

FRAME NO. 177

Fig. 2. Graphical Computation for U.S.S. HAMILTON
Method.

**2**



U.S.S. HAMILTON

88 1/2

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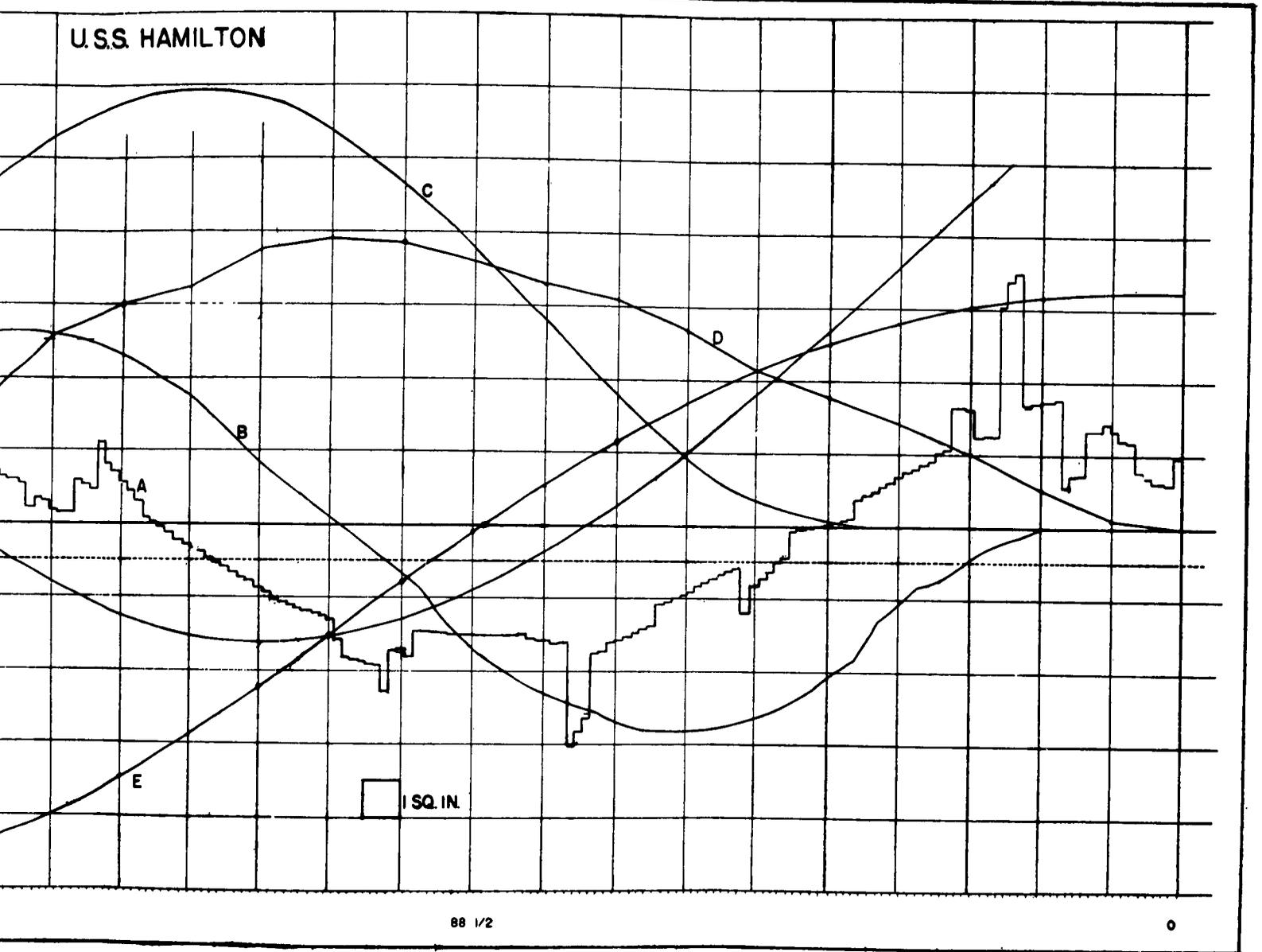
cal Computation for U.S.S. HAMILTON by Schadlofsky's

equivalent end value (according to Taylor) is $\frac{14.03}{1.608}$ = 8.73 in. (.608 being the ratio of center to end deflection of a free-free uniform bar). Applying the scale factor this gives for the end value: Ey = 8.73 x 228.7 = 1997 ton ft./in.[a] Taking E as 13,300 tons/in.[a] (equivalent to 2.1 x $10^6$ kg/cm[a] used by Schadlofsky) we get for the deflection at the forward perpendicular

$$y_{v.p.} = \frac{1997}{13300} = 0.1502 \text{ ft.}$$

hence, by Eq. (6)

$$n = \frac{1}{2\pi}\sqrt{\frac{g}{y_{v.p.}}} = \frac{1}{2\pi}\sqrt{\frac{32.2}{0.1502}} = 2.33 \text{ per sec.}$$

$$N_{th} = 140 \text{ per minute.}$$

In the computation by Schadlofsky's method (Fig. 2) the maximum value of the Ey curve is 13.15 in. The ratio of end to maximum value of the corrected amplitude curve is $\frac{1}{1.51}$ hence $Ey_{v.p.} = \frac{13.15}{1.51} = 8.70$ in.

The scale factor is 1 in. = 200.2 ton ft./in.[a]

Hence

$$y_{v.p.} = \frac{8.70 \times 200.2}{13300} = 0.131 \text{ ft.}$$

$$n = \frac{1}{2\pi}\sqrt{\frac{g}{y_{v.p.}}} = \frac{1}{2\pi}\sqrt{\frac{32.2}{0.131}} = 2.50 \text{ per sec.}$$

$$N_{th} = 150 \text{ per minute.}$$

The difference between the values computed by the two methods is due to the non-fulfillment of the end conditions by Taylor's method. The residual shear obtained by integration of the first dynamic load curve is 15 per cent of the weight of the ship. With a residual of such a magnitude, joining the ends of the shear curve causes considerable error. In general it may be said that Taylor's simplified method will give too low a value for the natural frequency if the residual shear exceeds about 4 per cent of the weight.

The tabular method of F. M. Lewis (7) offers a convenient substitute for the graphical methods. In this method the integrations are carried out by computation using the trapezoidal rule, dividing the ship into twenty parts. The method involves the initial assumption that the amplitudes vary as in a free-free uniform bar and these amplitudes are later corrected to satisfy the end conditions. After the corrected amplitude function is obtained the frequency is computed by the energy method. The kinetic energy and potential energy are summed up for the twenty sections and equated. Where the weight and moment of inertia curves are not too irregular this method should give as accurate results as Schadlofsky's. For very irregular cases the same method could be worked out with a greater number of stations. The method involves considerably less labor than the graphical methods

Table I   Computation for U.S.S. HAMILTON by Lewis' Tabular Method

| 1 | 2 x | 3 w₁ | 4 w₁x | 5 w₁x² | 6 f(x) | 7 w₁f(x) | 8 w₁xf(x) | 9 Ax | 10 y₁ | 11 y₁² | 12 w₁y₁ | 13 1/Iₘ | 14 2 | 15 1/Iₘ 2 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 10 | .53 | 5.30 | 53.00 | 0.0 | 0.0 | 0.0 | .0546 | -.6656 | .4430 | .2348 | .291 | 0.0 | 0.0 |
| 3 | 9 | 2.23 | 20.07 | 180.63 | .141 | .314 | 2.830 | .0491 | -.5301 | .2810 | .6266 | .448 | .0015 | .0007 |
| 5 | 8 | 5.40 | 43.20 | 345.60 | .288 | 1.555 | 12.442 | .0437 | -.3885 | .1509 | .8149 | .534 | .014 | .0075 |
| 7 | 7 | 5.69 | 39.83 | 278.81 | .428 | 2.435 | 17.047 | .0382 | -.2540 | .0645 | .3670 | .664 | .061 | .0405 |
| 9 | 6 | 5.35 | 32.10 | 192.60 | .561 | 3.001 | 18.008 | .0328 | -.1264 | .0160 | .0856 | .823 | .152 | .1251 |
| 11 | 5 | 2.38 | 11.90 | 59.50 | .684 | 1.628 | 8.140 | .0273 | -.0089 | .0008 | .0019 | .976 | .295 | .2879 |
| 13 | 4 | 8.04 | 32.16 | 128.64 | .791 | 6.360 | 25.439 | .0218 | .0926 | .0086 | .0691 | 1.073 | .476 | .5107 |
| 15 | 3 | 5.45 | 16.35 | 49.05 | .879 | 4.791 | 14.372 | .0164 | .1752 | .0307 | .1673 | 1.086 | .669 | .7265 |
| 17 | 2 | 7.55 | 15.10 | 30.20 | .946 | 7.142 | 14.285 | .0109 | .2367 | .0560 | .4228 | 1.091 | .831 | .9066 |
| 19 | 1 | 6.76 | 6.76 | 6.76 | .986 | 6.665 | 6.665 | .0055 | .2713 | .0736 | .4975 | 1.106 | .958 | 1.0595 |
| 21 | 0 | 5.31 | 222.77 | 0.00 | 1.000 | 5.310 | 119.228 | 0.0 | .2798 | .0783 | .4158 | 1.055 | 1.000 | 1.0550 |
| 23 | -1 | 6.60 | -6.60 | 6.60 | .986 | 6.508 | -6.508 | -.0055 | .2603 | .0678 | .4475 | .975 | .958 | .9341 |
| 25 | -2 | 6.94 | -13.88 | 27.76 | .946 | 6.565 | -13.130 | -.0109 | .2149 | .0462 | .3206 | .893 | .831 | .7421 |
| 27 | -3 | 6.09 | -18.27 | 54.81 | .879 | 5.353 | -16.059 | -.0164 | .1424 | .0203 | .1236 | .804 | .669 | .5379 |
| 29 | -4 | 6.01 | -24.04 | 96.16 | .791 | 4.754 | -19.016 | -.0218 | .0490 | .0024 | .0144 | .757 | .476 | .3603 |
| 31 | -5 | 8.54 | -42.70 | 213.50 | .684 | 5.841 | -29.207 | -.0273 | -.0635 | .0040 | .0342 | .618 | .295 | .1823 |
| 33 | -6 | 2.16 | -12.96 | 77.76 | .561 | 1.212 | -7.271 | -.0328 | -.1920 | .0369 | .0797 | .470 | .152 | .0714 |
| 35 | -7 | 2.69 | -18.83 | 131.81 | .428 | 1.151 | -8.059 | -.0382 | -.3304 | .1092 | .2937 | .376 | .061 | .0229 |
| 37 | -8 | 3.19 | -25.52 | 204.16 | .288 | .919 | -7.350 | -.0437 | -.4759 | .2265 | .7225 | .222 | .014 | .0031 |
| 39 | -9 | 2.48 | -22.32 | 200.88 | .141 | .350 | -3.147 | -.0491 | -.6283 | .3948 | .9791 | .156 | .0015 | .0092 |
| 40 | -10 | .63 | -6.30 | 63.00 | 0.0 | 0.0 | 0.0 | -.0546 | -.7748 | .6003 | .3782 | .068 | 0.0 | 0.0 |
| Sums | | 100.02 | -191.42 / 31.35 | 2401.23 | | 71.855 | -109.747 / 9.481 | | | | 7.0968 | | | 7.5743 |

$$y_1 = f(x) + A + B = (6) + (9) + B$$

$$I_m = 243 \text{ ft}^4; \quad L = 310 \text{ ft.}; \quad \Delta = 1382 \text{ ton} \quad N = 1{,}859{,}000 \sqrt{\frac{7.5743}{7.0968}} \sqrt{\frac{243}{1382 \times 310^3}}$$

$$N = 148 \text{ per minute}$$

$$A = \frac{71.86 \times 31.35 - 9.48 \times 100.02}{100.02 \times 2401.23 - (31.35)^2} = 0.00546$$

$$B = \frac{71.86 \times 2401.23 - 9.48 \times 31.35}{(31.35)^2 - 100.02 \times 2401.23} = -0.7202$$

Note: In column 13 the mean moment of inertia over each station was used instead of the value at the center of the station.

and has the advantage that an integraph is not required. A computation for the U.S.S. HAMILTON by this method is given in Table I. For the definition of terms used in this table reference should be made to ref. (7).

### b. Correction for Effect of the Surrounding Water

The largest single correction to the theoretical frequency is that due to the effect of the surrounding water. J. B. Henderson (9) and other early investigators, in the light of the theory of damped oscillations, considered that the water had a negligible effect upon the natural frequency.

Apparently the first experimental investigation of this question was undertaken by H. W. Nicholls (10) in 1924 by model experiments. Nicholls vibrated steel strips 30 in. x 2 in. to which were fastened wooden blocks, the exciting impulses being produced by electro-magnets. The natural frequencies were measured in air and in water, the models being supported at the computed free-free nodes. Nicholls found that the effect of the water was to lower the natural frequency considerably and that this effect could conveniently be expressed in terms of the increase in mass which would produce the same change in the natural frequency, the so-called "virtual mass". Hence the virtual mass was defined by the equation

$$\frac{\text{frequency in water}}{\text{frequency in air}} = \sqrt{\frac{D}{D + V.M.}} \quad \cdots \cdots \cdots \cdots (9)$$

where D is the displacement, and V.M. is the virtual mass.

Nicholls found that for a rectangular model of breadth b and depth d the relation between virtual mass and displacement could be expressed as

$$\frac{V.M.}{D} = 0.37 \frac{b}{d} + 0.20 \quad \cdots \cdots \cdots \cdots \cdots \cdots \cdots (10)$$

the ratio being in this case 0.78. For a triangular model he found,

$$\frac{V.M.}{D} = 0.70$$

In 1928 E. B. Moullin and A. D. Browne (11) conducted similar experiments with solid steel bars, a method which seems scarcely applicable to the ship problem because of the density. They did, however, make a full-scale test of a racing eight, 60 feet long and weighing 280 lb., supporting it out of water at the nodes. They found the natural frequency in water to be 54 per cent of its value in air which represents a virtual mass of 243 per cent of the displacement.

The first theoretical treatment of the water effect for actual ships is due to F. M. Lewis (12). In his theory the water vibrating with the hull at any section is considered to be one half the quantity of water which would move with the section and its image if submerged in a liquid of infinite volume, this problem being capable of a theoretical solution for certain forms approximating ship forms. The ship is divided into a convenient number of sections for each of which the appropriate form is to be used. A correction factor is introduced to allow for the

fact that the motion of the water is not confined to transverse planes. Lewis gave
the values for a number of forms similar to ship forms. The virtual mass having
been computed for as many sections as desired from Lewis' curves, these increments
are to be added to the weight curve of the ship after which Taylor's or other
methods are to be applied in the usual manner. It is to be noted that the correc-
tions will not be the same as would be obtained by adding up the virtual mass of
all sections and applying the water correction in one lump by formula (9). The
distribution of the water effect must be taken into account.

The logic of extending the hydrodynamic problem of the water moving with a
continuously moving submerged body to the case of vibratory motion may be open to
question. Schadlofsky (6) has criticised Lewis in that his method shows the virtual
mass to be independent of frequency and amplitude. However neither of these con-
siderations is likely to be of importance in the frequencies encountered in ship
vibration. Lewis' method is not confirmed by high frequency model experiments made
at the U. S. Experimental Model Basin to be discussed later. On the other hand
F. H. Todd (8) has applied Lewis' virtual inertia values to a number of ships for
which the actual frequencies were measured. Of 13 examples the average error is
3.9 per cent, rejecting the maximum error of 44 per cent.

Schadlofsky (6) regards the water effect as a damping action which may be
expressed as an equivalent increase in weight by the formula

$$W = G \left( \frac{n_1^2}{n_w^2} - 1 \right) \quad \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots (9a)$$

which is identical with the virtual mass formula (9) given above. In Schadlofsky's
formula $W$ is the damping resistance, $G$ is the weight of the model or ship, $n_1$ is
the frequency in air, and $n_w$ the frequency in water.

Schadlofsky takes the view that the water effect cannot be determined by
mathematical analysis but must be determined by model measurements and that in
going from model to full scale ship the laws of dynamic similarity must be taken
into account. He shows that dynamic similarity between model and ship cannot be
attained with respect to both skin friction and resistance due to flow, but that
for the same Froude's number the resistance due to flow for the ship can be de-
termined from the model and that the neglect of the resistance due to skin friction
will cause but a small error.

Schadlofsky attempts to show that the damping resistance can be estimated
with sufficient accuracy for most practical cases from data he has collected on a
number of rectangular models and models of ship form. For this purpose he intro-
duces the non-dimensional damping index $K_w = W/l^4 n^2 \rho$ where:

    $W$ is the actual damping resistance
    $l$ is any linear dimension of the model

n is the frequency

$\rho$ is the mass density

Schadlofsky's experimental method was similar to Nicholl's, the models consisting of steel strips to which were fastened wooden blocks with a covering of rubber between the blocks. The models were vibrated by means of electromagnets and were supported at the computed nodes in air and in water.

The experiments showed that the damping resistance was a function of frequency contrary to Lewis' theory. For ship forms the draft was found to be of considerable influence on the damping resistance as was also the position of the nodes. The damping resistance was found to be minimum when the nodal position corresponded to that of the free-free bar.

A recent study of this problem by J. J. Koch (13) who used an electrical analogy is of academic interest and has not found practical application. A. Dimpker (14), who set various forms in forced vibration in water, found the virtual mass to be a function of draft and frequency.

Exact experimental verification of the various methods of computing the water effect on full size ships is lacking because it is impossible to isolate the water effect from other effects. Pallograph records of free vibrations on the U.S.S. HAMILTON, induced by dropping the anchor, indicated that the logarithmic decrement was 0.023. This low value is in agreement with measurements made by J. L. Taylor (24), and indicates that the damping must be small and the inertia effect predominant.

For a rough estimate it may be assumed that the effect of the water is to lower the frequency by 25 per cent. For greater accuracy the theoretical method of F. M. Lewis (12) or the empirical data of E. Schadlofsky (6) may be used.

From Schadlofsky's data it is estimated that the frequency of the U.S.S. HAMILTON is lowered by 17 per cent due to water effect. From the data in Taylor's first paper (5) this estimate would have been 16 per cent. In Lewis' method for water correction the effect cannot be isolated since the virtual mass is added directly to the weight of the ship. To find the water effect separately the computation must be repeated with the virtual mass omitted.

c.  Correction for Effect of Elastic Behavior.

The correction that must be applied to $N_{th}$ for the variation in elastic behavior of a ship from that of a slender beam is as controversial a matter as that of the water effect although the magnitude of the correction is less.

The first of the elastic effects to be taken account of in vibration theory is the deflection due to shear, which for the slender beam is negligible in proportion to the bending deflection. As the shear deflection indicates a decrease in rigidity this effect tends to lower the natural frequency.

A further effect is the increase in deflections of boxlike structures as

compared with simple beams. There is also the effect of rotary inertia of the superstructure which causes the frequency to be lower than it would be if all points executed merely a vertical simple harmonic motion.

In previous methods it was the practice to use a reduced value of the elastic modulus called the effective modulus of structure to allow for the combined effects, but as they came to be better understood it seemed preferable to treat them separately.

To allow for elastic behavior Taylor corrected the theoretical frequency by the formula

$$N = \sqrt{\frac{N_{th}}{(1 + r)(1 + r')(1 + r'')}} \quad \ldots \ldots \ldots \ldots (11)$$

where r is the ratio of shear deflection to bending deflection, r' is the ratio of rotational kinetic energy to total kinetic energy, and r'' is the ratio of virtual inertia of the surrounding water to the inertia of the ship.

Schadlofsky likewise amended the theoretical frequency obtained from the graphical computation by a number of correction factors using the formula:

$$N = \sqrt{\frac{N_{th}}{k_1 \cdot k_2 \cdot (1 + k_3)(1 + k_4)(1 + k_5)}} \quad \ldots \ldots (12)$$

here $k_1$ = ratio of total moment of inertia to effective moment of inertia

$k_2$ = ratio of actual deflection to deflection calculated from flexure theory

$k_3$ = ratio of shear deflection to bending deflection multiplied by an empirical constant dependent upon the number of decks

$k_4$ = ratio of rotational kinetic energy to total kinetic energy

$k_5$ = ratio of damping resistance to weight of ship

With this formula the value of E to be used is 2.1 x $10^6$ kg/cm² = 30 x $10^6$ lb/in². Sufficient data are supplied in Schadlofsky's paper for estimating the various k's. In the case of the U.S.S. HAMILTON these values are as follows:

$k_1$ = 1.015; $k_2$ = 1.050; $k_3$ = .075; $k_4$ = .032; $k_5$ = .436

Applying the above correction factors to the theoretical frequencies of five ships Schadlofsky was able to check measured values with a maximum error of 1.5 per cent.

The corrections of Schadlofsky should be applicable to the theoretical computation made by any of the methods discussed as they are all based on the same initial assumptions.

Comparative figures for the U.S.S. HAMILTON computed by the three methods discussed are shown in the following table. Lewis' virtual mass was omitted from the computations and hence the theoretical frequencies are comparable.

## Table II
### U.S.S. HAMILTON - COMPUTATION OF TWO-NODED VERTICAL FREQUENCY

| | Taylor's Simplified Method | Schadlofsky's Method | Lewis' Tabular Method |
|---|---|---|---|
| Theoretical frequency ($N_{th}$) | 140 | 150 | 148 |
| Correction factor for water effect | $\dfrac{1}{\sqrt{1.40}}$ | $\dfrac{1}{\sqrt{1.436}}$ | |
| Correction factor for shear deflection | $\dfrac{1}{\sqrt{1.115}}$ | $\dfrac{1}{\sqrt{1.075}}$ | |
| Correction factor for rotary inertia | $\dfrac{1}{\sqrt{1.032}}$ | $\dfrac{1}{\sqrt{1.032}}$ | |
| Correction factor for effective moment of inertia | | $\dfrac{1}{\sqrt{1.015}}$ | |
| Correction for variation of actual from theoretical deflection with loading | | $\dfrac{1}{\sqrt{1.050}}$ | |
| Value of E used (lb.per sq.in.) | $30 \times 10^6$ | $30 \times 10^6$ | $30 \times 10^6$ |
| Computed frequency | 110 | 115 | |
| Experimental frequency | 107 | 107 | 107 |
| Error | +3% | +7½% | |

[Note: In reference 28 the value of $N_{th}$ obtained by Taylor's method was reported as 147 per minute and the computed frequency as 116 per minute. Due to an error in reading the scale of the integraph these values were incorrect and the values given in the above table should be substituted]

While the error in the computed frequency is less in this case by Taylor's simplified method than by Schadlofsky's, it is believed that the value for the theoretical frequency computed by the latter method is more accurate and that the errors are due to the correction factors.

While thus the theoretical computations of the vertical two noded frequency gives results in fair agreement with experimental values, the data required for the computation are usually not readily available and the simple Schlick formula is preferable where the degree of accuracy required will permit.

Where the theoretical computation seems desirable the tabular method of Lewis is sufficiently accurate and fulfills both end conditions. In using this method the water effect is best allowed for by adding the virtual mass of each section to the weight using Lewis data (12). This, however, will not give the

theoretical frequency separately. The effect of elastic behavior may be allowed for
by using either Taylor's or Schadlofsky's data, omitting the water correction factor
in this case. An alternative to the above procedure is to use Lewis' tabular method
without including the virtual mass and to apply to the results, which in this case
is the theoretical frequency, formula (11) or (12). In lieu of the correction for-
mula here the actual frequency can be closely approximated by reducing the theo-
retical frequency by 30 per cent.

A further discussion of the relative merits of Schlick's formula and the the-
oretical computations is given in sections 6 and 7.

### 4. Other Types of Vibration

In general two-noded horizontal vibration can be treated in the same manner
as two-noded vertical vibration, and was provided for in the Schlick formulas. H.
W. Nicholls (10) applied his energy balance method to horizontal vibration in the
same way as to vertical vibration. J. L. Taylor (5) points out that interaction
between horizontal and torsional vibration may occur when the center of mass of the
section does not coincide with the center of area. The computation of horizontal
frequencies requires a knowledge of the moment of inertia about a vertical axis.

The first study of torsional vibration of ships apparently was made by L.
Gumbel (16) who applied to the ship the theory of torsional vibration of shafts. A
graphical method was worked out by H. W. Nicholls (10) based on Lewis' method for
irregular shafts (17). In this method the equivalent shaft is considered to have a
continuously varying mass moment of inertia whereas Gumbel assumed a number of con-
centrated masses attached to the shaft each representing a section of the ship. F.
Horn (18) found as did Nicholls that the effective polar moment of inertia of the
section was considerably lower than the calculated. Recognizing that an accurate
calculation of the torsional frequencies is seldom practical because of the amount
of labor involved Horn offered the simple formula:

$$N = 60c \sqrt{\frac{g \ G \ J_{to}}{D \ i_o^2 \ L}} \quad \dots \dots \dots \dots \dots \dots \dots \dots (13)$$

where  N   = frequency

      g   = acceleration of gravity

      G   = shear modulus of elasticity

      $J_{to}$ = polar moment of inertia of midship section

      D   = displacement

      $i_o$  = radius of gyration of total weight with respect to the centroidal axis

      L   = overall length of ship

      c   = empirical coefficient.

c can be obtained only from data on similar ships and will vary for the same ship
with change in loading.

Taylor's method for torsional vibration (5) as for flexural vibration is to assume an amplitude profile. In this case the graphical method requires only two integrations. The calculated rigidity of the section is reduced by an empirical factor derived from Vedeler's investigation of the torsion of ships (19). Taylor as well as Nicholls excludes internal fluids in computing mass moment of inertia. Taylor points out also that the axis of torsion is not the centroid and that the stresses due to torsion are not proportional to the distance from any one point.

With regard to the higher harmonics it may be said that theory has not been developed up to the present time to the point where harmonics higher than the three noded type can be computed with any degree of accuracy. As the number of nodes increases the ratio of the span between nodes to the depth and beam becomes so small that the flexure theory of beams does not hold even approximately. The only guide to the estimate of frequencies above the three noded type is a comparison with measured values for similar ships. Moreover at the higher frequencies it is difficult to tell whether a vibration of the complete hull structure is taking place or whether the effect is due to a local resonance of various parts of the structure.

## 5. Vibration Investigations at the U.S. Experimental Model Basin

### a. Model Experiments:

An attempt was made at the U.S. Experimental Model Basin to study the effect of the water on the natural frequency by model experiments. In these model experiments ship conditions were simulated as far as possible both as to the construction of the models and as to the method of inducing vibration.

Two models were made of sheet steel one rectangular and the other triangular in cross section. The details of construction are shown in the drawings, Figs. 3 and 4. For both models the length is 18 ft., beam 27 in., and depth 17½ in., giving the ratios: L/B = 8.0; L/D = 12.3; B/D = 1.54. The section is uniform over a length of 15 ft. with pointed ends extending 1½ ft. The material used is a sheet steel of nominal thickness 0.050 in. except for the outer deck plates which are 0.079 in. The elastic modulus of the material averaged $29 \times 10^6$ lb/in². Spot welding was used throughout except that the upper flanges of the transverse bulkheads were bolted to the deck. The triangular model has two transverse stiffeners between bulkheads which were not included in the rectangular model. It was found necessary to reinforce the deck of the triangular model between bulkheads with wooden braces to eliminate local vibrations.

For setting up vibration a small vibration generator manufactured by Losenhausenwerk was used. This machine, shown in the photograph Fig. 5, consists of two parallel shafts permanently geared so as to rotate in opposite directions, each shaft carrying the armature of a d.c. shunt motor. On the ends of the shafts are adjustable weights by means of which various degrees of unbalance can be
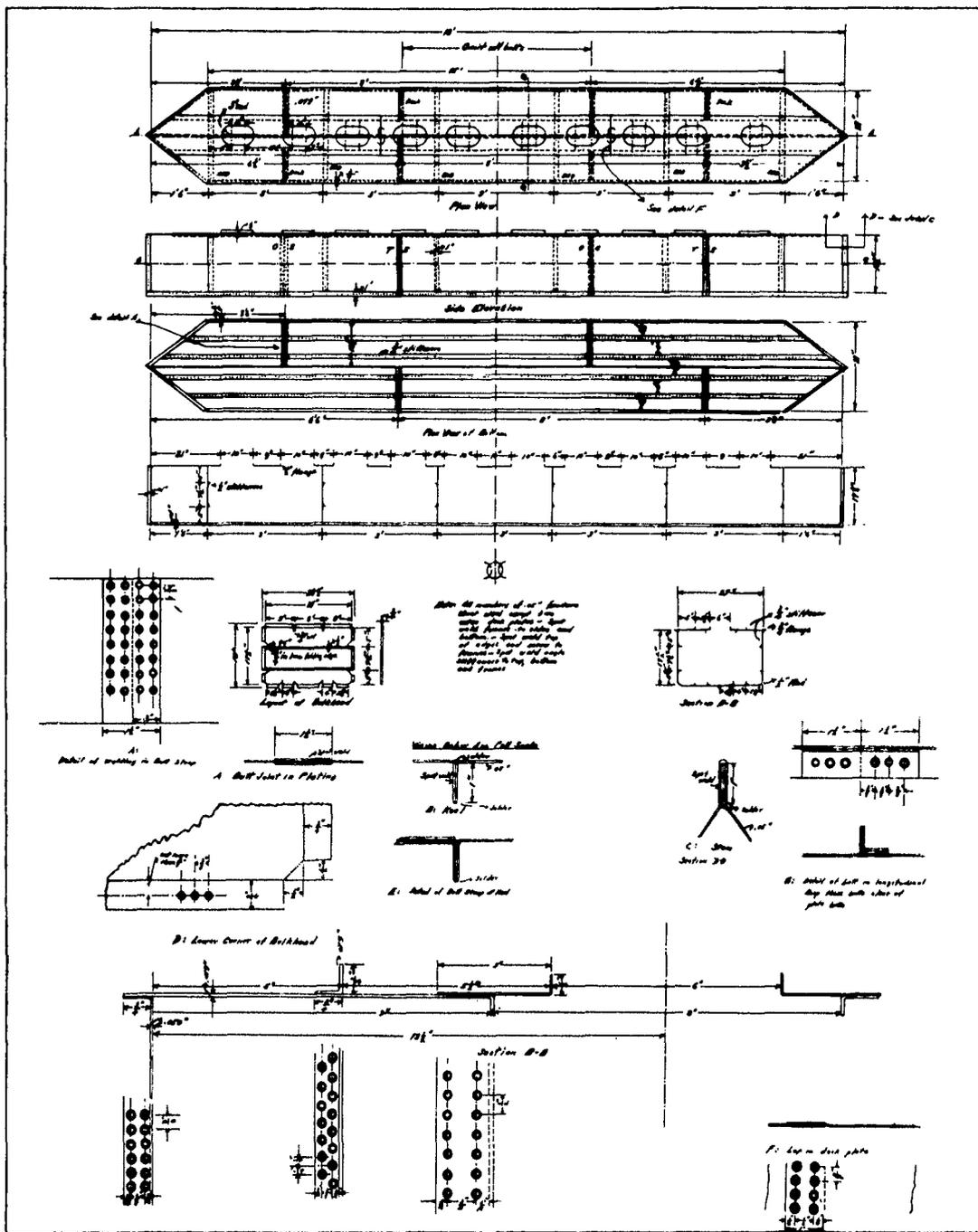
Fig. 3. Plan of Rectangular Model.

Fig. 4.  Plan of Triangular Model.

produced setting up sinusoidal forces or force moments. According to the manner of setting the weights vertical, torsional, or tilting oscillations can be produced. A precision tachometer gives the speed of the shafts and hence the impressed frequency. In using this machine the convenient unit of angular velocity is the Hertz, or one revolution per second.

The models were loaded with steel weights each 2 in. x 4 in. x 12 in. weighing approximately 28 lb. In a preliminary test the rectangular model was uniformly loaded to a total load of about 3000 lb. and set upon two rollers resting on a concrete floor, the rollers being placed under the theoretical nodes. The vibration generator was set in a frame which was clamped to the deck as shown in the photograph, Fig. 6. Upon running the machine it was found that the weights would not stay fixed in position due to the high acceleration and this made it impossible to produce a condition of resonance. It was considered unsatisfactory to bolt the weights to the model because the increase in stiffness would have raised the frequency beyond the capacity of the machine. This would also have made theoretical computations difficult because of the unknown change in stiffness. It was therefore decided to pour in a layer of marine glue so as to immerse the weights to a depth of about one inch. After doing this resonance could be produced but it was found impossible to produce a symmetrical vibration. The slightest adjustment intended to level up the model would shift the vibration from one end to the other.

To overcome this condition a steel framework was made from which the model was syspended by 72 springs as shown in the photograph, Fig. 7. The springs were calculated to make the natural frequency of the model as a whole on this suspension about 100 per minute which was far below the flexural frequencies to be investigated. This method of supporting the model out of water eliminated the uncertainty as to the proper location of the nodal points and closely simulated a free-free bar. Since over 100 free vibrations could be counted after giving the model a deflection it was apparent that the springs introduced very little damping.

The practical working range of the vibration generator was found to be from 900 to 2700 vibrations per minute or from 15 to 45 Hertz. The lower limit was determined by the unsteadiness of running of the machine at high eccentricities. The upper limit was determined by overheating of the controls. In order to obtain resonance curves it was necessary not only that both the resonance in air and water lie within this range but that they lie near the center of this range. The most favorable loading for both models was found to correspond to a draft of about 16 inches which was about the limit of their capacity. The only two favorable distributions of load were the uniform distribution and the condition where the load was concentrated at the center and the two ends. Resonance could not be produced with all the load concentrated at the ends or in the center.

The investigation was confined accordingly to the four following cases:
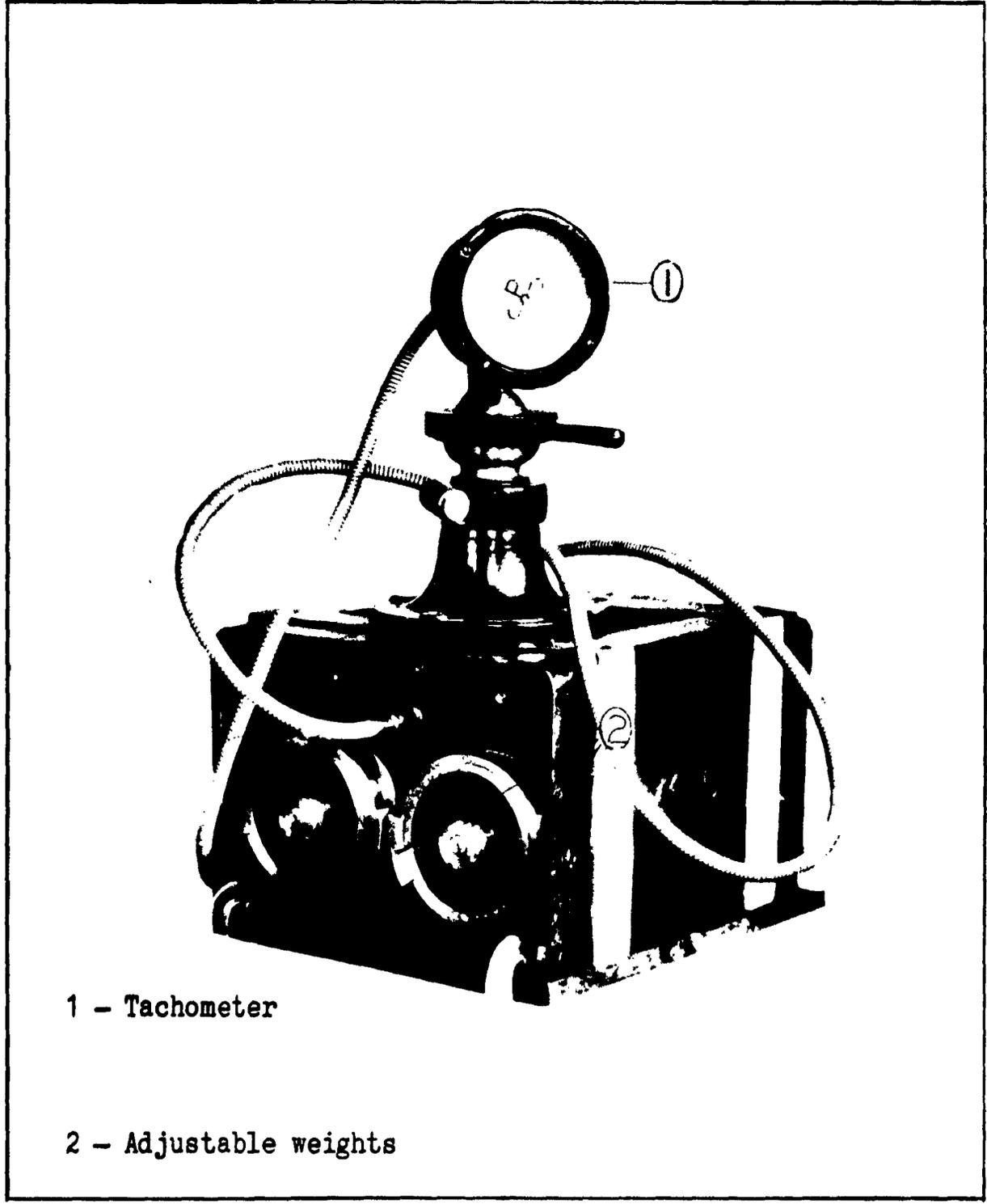
1 — Tachometer

2 — Adjustable weights

Fig. 5.   Small Vibration Generator.
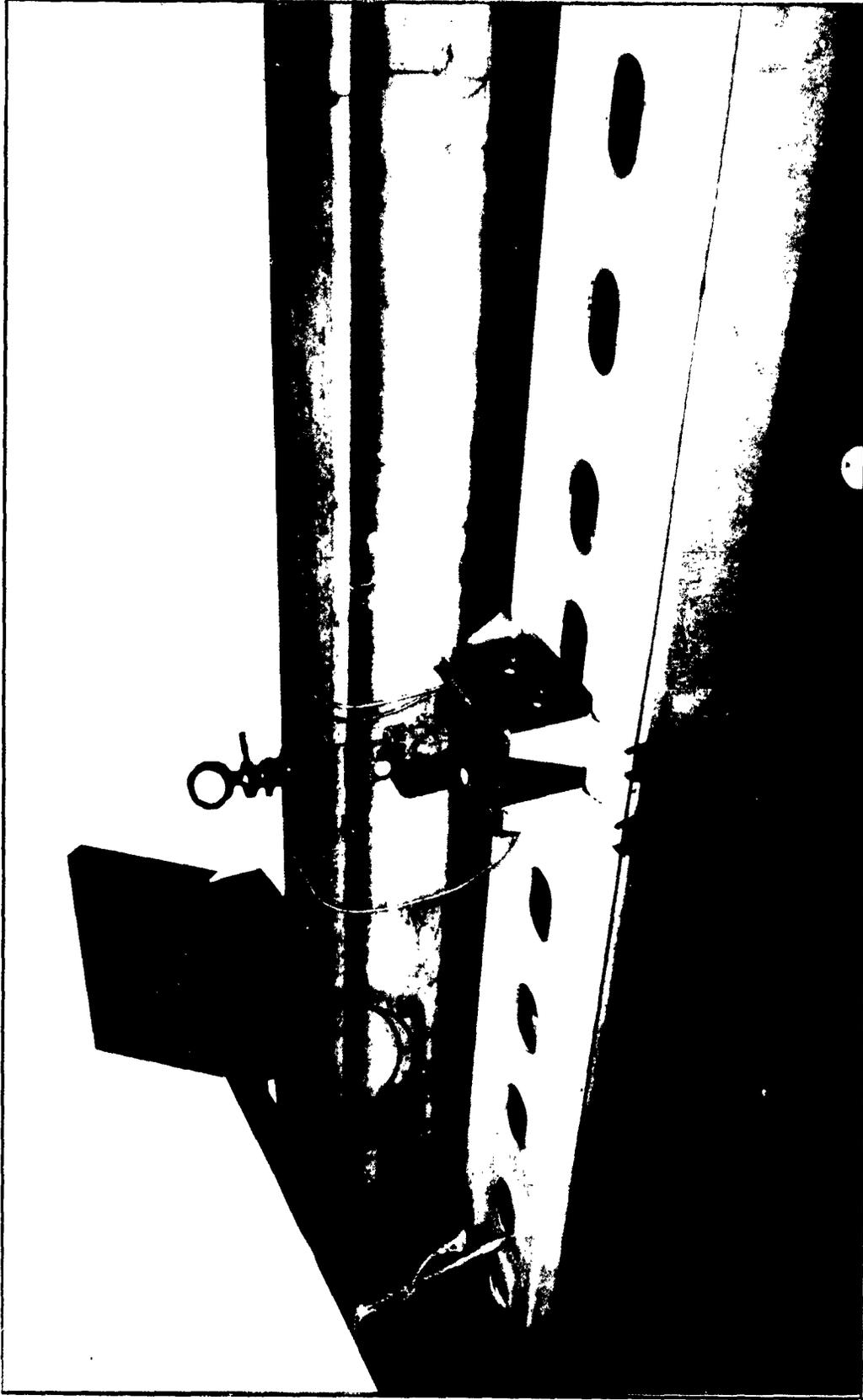
Fig. 6. Rectangular Model Ready for Test.

Note: The springs were
enclosed in wooden cases
to eliminate lateral vibration
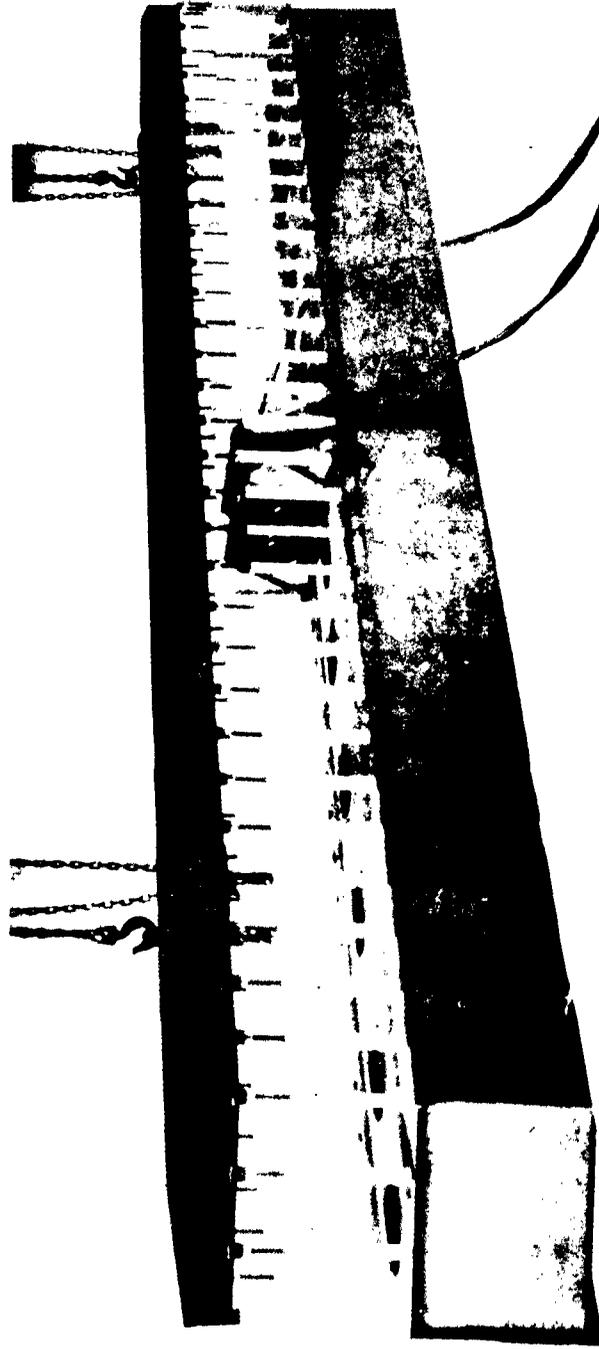
Fig. 7. Rectangular Model on Spring Suspension.

Uniform load and load concentrated at three points for both the rectangular and triangular models. For each case measurements and computations were made of the natural frequency in and out of water.

Amplitudes were read by means of a vibrograph consisting of an inertia element suspended by springs against which the stem of a dial gage pressed. This is shown in Fig. 6. The natural frequency of the element was about 2 Hertz. It was found that amplitudes less than 0.015" could be read satisfactorily within the range 15 to 45 Hertz.

The experimental procedure was as follows: The model having been properly loaded was suspended from the steel frame by the springs. The eccentricity of the vibration generator was set at the smallest value that would produce a distinct resonance. Amplitudes were read at the center of the model which in this case required placing the amplitude meter on top of the vibration generator itself. The machine was then run with gradually increasing speed while the amplitudes were recorded up to the highest speed at which readings could be obtained. Readings were continued as the speed was gradually decreased. If the data revealed a definite resonance the generator was then run at this speed and a series of amplitude readings was taken along the length of the model. Fig. 8 shows a typical resonance curve and Fig. 9 an amplitude profile. In attempting to locate the nodes it was found simplest to hold one of the dial gages in the hand, the inertia of the gage itself being sufficient at these high frequencies to give an indication of the point of minimum amplitude. The exact position of the nodal points, however, could not be determined within less than $\pm$ 3 inches. These data having been recorded with the model on the spring suspension, the springs were removed and the model lowered into the water whereupon the whole process was repeated with the model floating freely. The basin at this point was 10 ft. wide and 4 ft. 2 in. deep.

For each of the cases mentioned graphical computations were made using the methods of J. L. Taylor (5) and E. Schadlofsky (6). Computations were also made by the uniform bar formula for those cases closely simulating uniform loading. Shear deflections were computed for both models by the theoretical formula

$$y_s = 1/G \int \left[ \ Q/I^2 \ \int \ m^2/t \ \ ds \right] dx$$

where   G   = shear modulus

       Q   = total vertical shear

       I   = moment of inertia of entire section about neutral axis

       m   = moment about the neutral axis of the area between the point under consideration and the center line of top deck

       ds = element along the   shell measured from top deck center
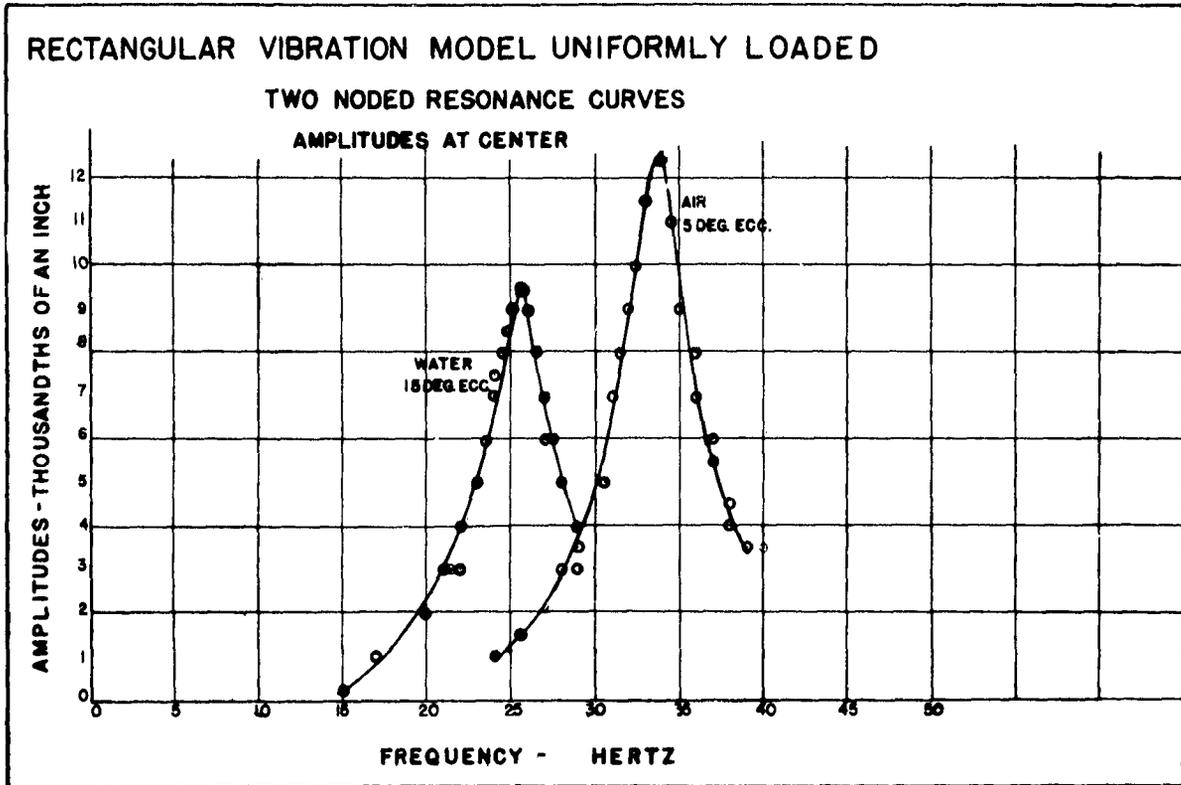
**Fig. 8. Resonance Curves for Rectangular Model.**
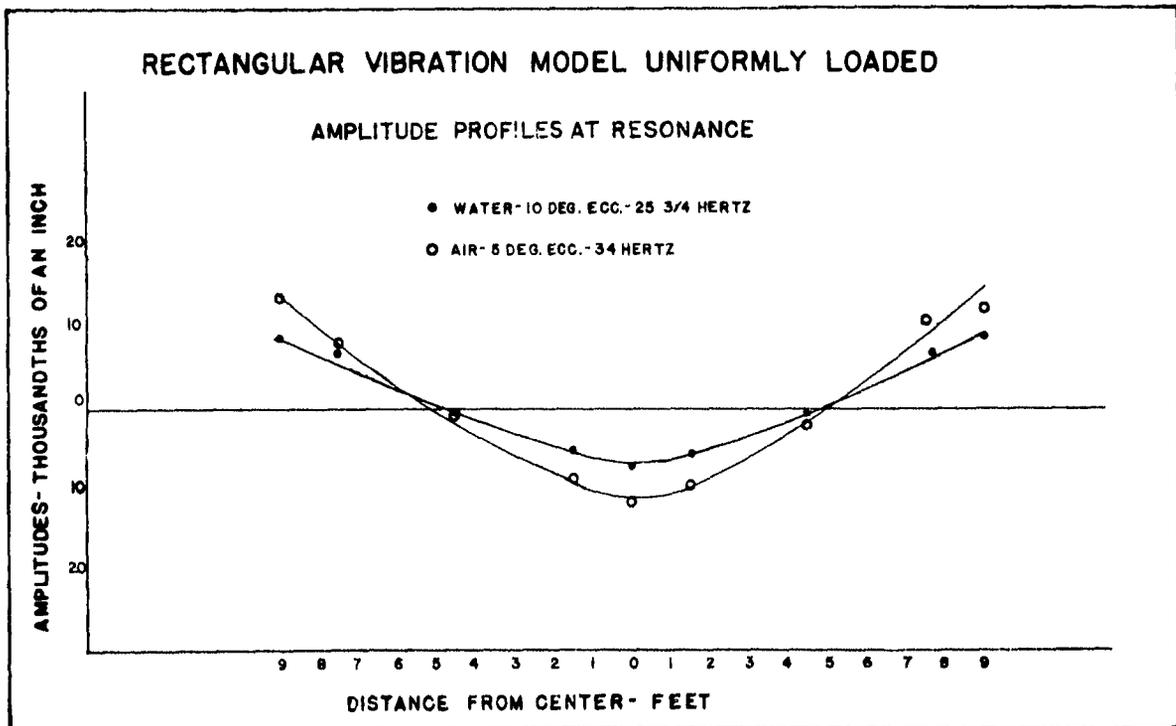


**Fig. 9. Amplitude Profiles for Rectangular Model.**

and the effect of shear was included in the computation of the natural frequencies. For discussion of shear deflection see reference 29. In applying Schadlofsky's method to the models, corrections for effective width and elastic behavior were omitted and the shear correction factor was derived from the above formula. However, for completeness, the correction factor including all the effects studied by Schadlofsky is given in next to the last line of Table IV below, and here the shear factor as well as the other factors are the same for both models. The theoretical water correction method of F. M. Lewis was applied to the graphical computation in both of the non-uniformly loaded cases, and also directly in the uniformly loaded cases by the uniform bar formula with the virtual mass added as a lump sum. The results of the measurements and computations as well as other items of interest are given in the following tables.

Table III

Model Dimensions

|  | Rectangular Model | Triangular Model |
|---|---|---|
| Length (ft.) | 18 | 18 |
| Beam (in.) | 27 | 27 |
| Depth (in.) | 17½ | 17½ |
| Displacement (lb.) | 3034 | 1490 |
| Draft (in.) | 16 | 16 |
| Moment of Inertia of Section (ft$^2$ in$^2$) | 2.56 | 1.19 |
| Weight of hull (lb.) | 458 | 358 |
| Weight of ballast (lb.) | 2576 | 1132 |
| Number of bulkheads | 6 | 6 |
| Thickness of plating (in.) | 0.05 | 0.05 |
| L/B (Length/Beam) | 8.0 | 8.0 |
| L/D (Length/Depth) | 12.3 | 12.3 |
| B/D (Beam/Depth) | 1.54 | 1.54 |
| Beam/Draft | 1.69 | 1.69 |

Table IV

Measured and Computed Quantities (all frequencies expressed in Hertz)

| | Rectangular Model | | Triangular Model | |
|---|---|---|---|---|
| | uniform loading | concentrated loading | uniform loading | concentrated loading |
| Measured frequency in air | 34.0 | 32.8 | 39.0 | 33.0 |
| Measured frequency in water | 26.0 | 31.8 | 35.2 | 32.5 |
| Distance of nodes from center - air | 5.0 ft. | 5.0 ft. | 4.2 ft. | 4.7 ft. |
| " " " " " -water | 5.0 ft. | 5.0 ft. | 5.0 ft. | 5.0 ft. |
| Frequency in air computed by Taylor's method with correction for shear deflection | 39.0 | 32.2 | 39.2 | 31.9 |
| Frequency in air computed by Schadlofsky's method with correction for shear deflection | 39.0 | 33.0 | 39.2 | 32.5 |
| Frequency in air computed by Schadlofsky's method without correction for shear deflection | 41.2 | 34.5 | 40.4 | 33.5 |
| Natural frequency in air by uniform bar formula with correction for shear deflection | 40.4 | | 39.8 | |
| Ratio of shear to bending deflection computed | 0.084 | 0.094 | 0.059 | 0.063 |
| Virtual mass by Lewis' method (per cent of displacement) | 89 | 89 | 84 | 84 |
| Frequency in water computed by Taylor's method with Lewis' virtual mass added | 28.8 | 25.9 | 28.8 | 26.1 |
| Frequency in water computed by Schadlofsky's method with Lewis' virtual mass added | 29.1 | 27.3 | 29.1 | 26.5 |
| Per cent virtual mass by formula: $$VM = 100 \times \left( \frac{n_a^2}{n_w^2} - 1 \right)$$ | 71 | 6 | 23 | 3 |
| $\sqrt{k_1 \cdot k_2 \cdot (1 + k_3)(1 + k_4)}$ according to Schadlofsky | 1.12 | 1.12 | 1.12 | 1.12 |
| $n_{th}/1.12$ | 36.8 | 30.8 | 36.1 | 29.9 |

It is seen from Table IV that the measured frequencies in air check those computed by either Taylor's or Schadlofsky's method in three out of four cases. The probable explanation of the discrepancy in the case of uniform loading for the rectangular model is that the glue which in this case alone covered the whole bottom caused a damping effect due to internal friction. Fig. (10) illustrates the graphical computation for the triangular model by Schadlofsky's method.

Next it is to be observed that the effect of the water was much less in the case of concentrated loading than in the case of uniform loading for both models. This is consistent with the fact that the concentration of the ballast at the anti-nodes increases the flexural inertia of the ship while the water inertia is not greatly changed. Hence the damping action is relatively less for the case of concentrated loading. However, the fact that damping action for the case of concentrated loading causes less than a 3 per cent lowering of the frequency is surprising

The application of Lewis' virtual mass factors to the graphical computations did not predict the frequencies in water within 10 per cent for uniform loading and within 20 per cent for concentrated loading. However as previously mentioned the full scale measurements of F. H. Todd indicate that Lewis' method is more reliable at low frequencies.

Unfortunately the range of $F^{-1}$ values (reciprocal of Froude's number) investigated by Schadlofsky, namely, from $0.39 \times 10^{-2}$ to $3.85 \times 10^{-2}$, does not include our models which had an $F^{-1}$ value of about $0.15 \times 10^{-2}$. Schadlofsky's experiments showed definitely that the water effect increases with frequency, a fact ignored in Lewis' theory. On the other hand, the high frequencies used in the model experiments are not encountered in practice. The magnitude of Schadlofsky's corrections for elastic behavior is shown in the last two items of Table IV. Comparison of measured values in air with those computed with these corrections indicates that Schadlofsky's factors, at least for these models, are too high. Schadlofsky's curves give a ratio of shear to bending deflections of 0.17, whereas, the computed values range from 0.06 to 0.09.

The results at the Experimental Model Basin seem to indicate that the correction for shear deflection and the correction for water effect are all that need be taken into account in deducing the actual frequency from the theoretical frequency.

As the water effect necessitates the largest single correction to the theoretical frequency it would seem that without model measurements which in general are not feasible, the frequency cannot be forecast with an accuracy better than ±5 per cent by methods now in use.

### b. Full Scale Experiments

In full scale testing the U. S. Experimental Model Basin conducted measurements on four fleet oilers in 1929 (20) by the method of dropping the anchor. Recording was done with a Sperry pallograph. Also, at this time the natural

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LONGITUDINAL SCALE - 1 INCH = 1 FOOT
A - DYNAMIC LOAD - 1 INCH = 25 LB/FOOT
B - SHEAR - 1 INCH = 100 LB
C - BENDING MOMENT - 1 INCH = 400 FT LB
D - M/I - 1 INCH = 335 LB/INCH$^2$ FOOT
E - SLOPE - 1 INCH = 1340 LB/INCH$^2$
F - EY - 1 INCH = 5360 FT LB/ INCH$^2$

1 SQ. IN.

TRIANGULAR MODEL - CONCENTRATED LOAD
WITH VIRTUAL MASS ADDED - PARALLEL
SHIFT OF BASE PREVIOUSLY MADE
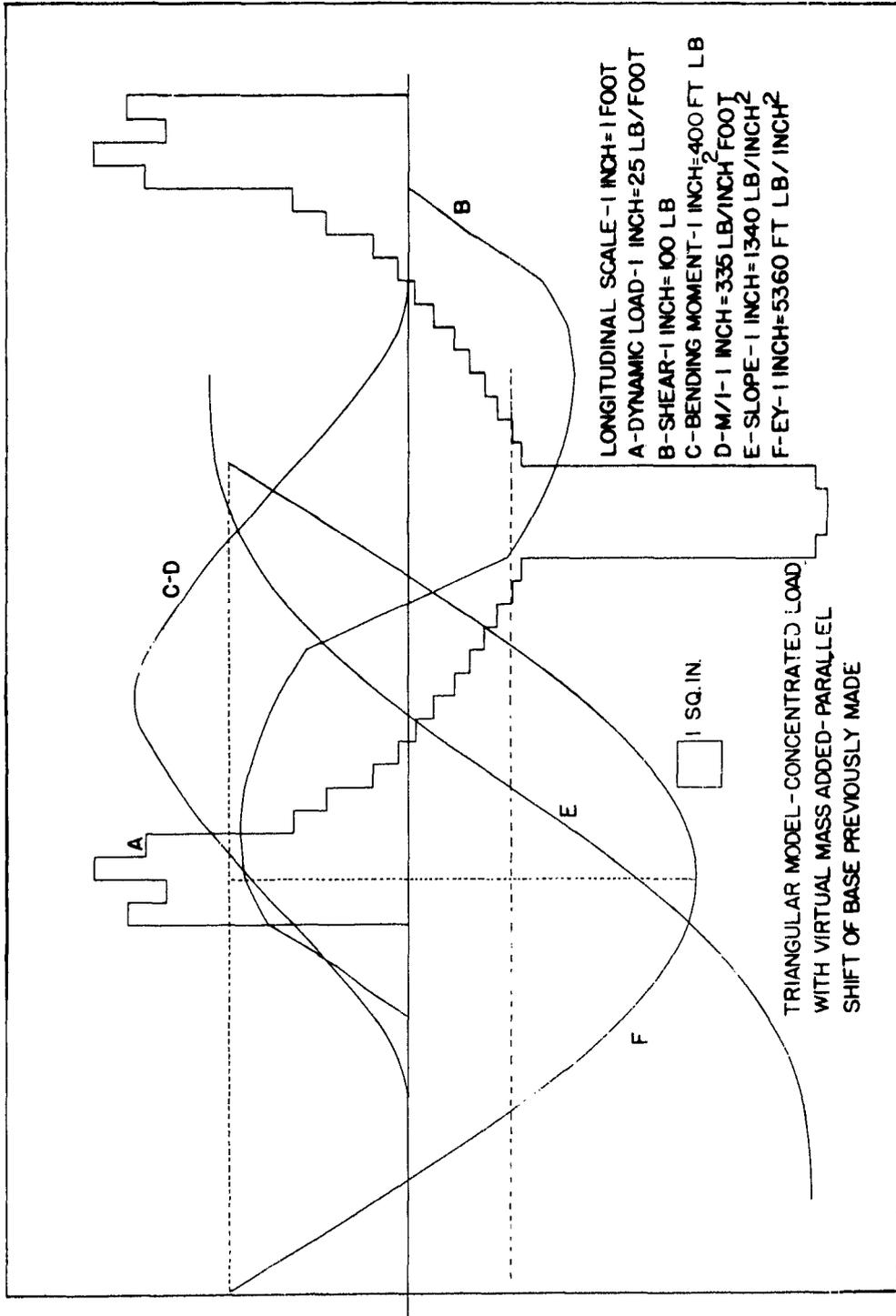
**Fig. 10. Graphical Computation for Triangular Model.**

frequency of the battleship OKLAHOMA was determined by running the engines in the region of resonance and recording the vibration with the pallograph. The anchor method was used in this case as a check. For the tanker CUYAMA and the battleship OKLAHOMA, computations were made by Taylor's method, by a variation of Lewis' tabular method, and by Taylor's method with Lewis' correction for virtual mass. For the CUYAMA, the frequency computed by Taylor's method with Lewis' correction for virtual mass was 5 per cent higher than the measured value. For the OKLAHOMA it was 19 per cent lower.

In 1932 the 25 ton vibration generator built by Losenhausenwerk of Düsseldorf, Germany, was acquired by the Model Basin. This machine is shown in the photograph, Fig. 11. A discussion of the use of this type of machine in bridge testing is given in ref. (21). Opportunity did not present itself to make a test with this machine until May, 1933, when it was installed on the destroyer HAMILTON. This test is described in ref. (28). The test showed considerable change in the water effect with depth of channel. The natural frequency increased from 92 to 107 per minute in going from a depth of 17 feet to a depth of 27 feet. The upper value was later checked by pallograph measurements at a considerably greater depth. The computation by the graphical method of J. L. Taylor with correction for shear and rotary inertia would have required the introduction of the water correction factor $\frac{1}{\sqrt{1.49}}$ to check the measured frequency if the correction were applied as a lump factor. The Schlick constant required to check the measured value would have been $1.16 \times 10^5$. The three noded resonance in "deep" water occurred at 206 vpm or 1.93 times the fundamental as against 2.76 for the uniform bar or 2.26 for bars with pointed ends, according to Schadlofsky. These figures might serve as a basis for estimating the two and three noded frequencies of other destroyers either by the Schlick formula or the graphical computation.

### c. Vibration Recording

Because of the low frequencies encountered in the study of hull vibration the recording instruments must be of different design from the usual instruments for stationary work. As this particular field of vibration has only recently received much attention, few instruments suitable for such measurements are as yet available. At the time of the test on the U.S.S. HAMILTON just mentioned, a suitable pallograph was not available at the Experimental Model Basin with the result that the only amplitude measurements made were those taken from shore by sighting with a surveyor's transit. Subsequently the Type "A" pallograph shown in the photograph, Fig. 12, was made. The inertia element which is suspended by a set of fulcrum springs has a natural frequency of 20 per minute so that it accurately records frequencies of the order of 100 per minute. This instrument was used to record vibration during one of the trials of the U.S.S. HAMILTON, March 26-7, 1934. Analysis of these records showed that vibration occurred both in the bow and in the

Fig. 11. Large Vibration Generator.

stern in the neighborhood of the two and three noded resonances previously determined with the vibration generator. In every case the frequency recorded was either the same as the shaft RPM or three times as much, indicating that impulses were being set up both by the propeller blades as well as by unbalance in the propellers themselves. The anchor was dropped in shallow water after which 34 vibrations were recorded by the pallograph at a frequency of 99 per minute as compared with 92 per minute measured previously in shallow water. From this record it was possible to estimate the logarithmic decrement of damping due to skin friction and internal friction discussed in section 3 (b).

These measurements indicated that the problem can be studied during ordinary runs without the necessity of installing the vibration generator and with this in view a more portable pallograph, the type "B" shown in the photograph, Fig. 13, has recently been designed.

## 6. Tabulation of Available Data on Ship Frequencies

For reference there are tabulated below the results of measurements already reported in technical publications. In cases where no information is available for making computations these figures may serve as an indication of the ranges in which resonance is likely to occur. The computed values were obtained by various methods and in many cases the good agreement with measured values is due to the arbitrary use of correction factors.

Table V

| Author and reference | Type of Vessel | Displacement (tons) | Type of Vibration | Frequency | |
|---|---|---|---|---|---|
| | | | | Meas. | Comp. |
| Todd (8) | Cargo | 7,940 | 2 node vert. | 100 | 109.5 |
| | Tanker | 15,190 | " | 78.9 | 74.2 |
| | " | 4,377 | " | 105-6 | 107.5 |
| | Cargo | 6,590 | " | 81-2 | 117.8 |
| | " | 3,835 | " | 109 | 112.2 |
| | Tanker | 4,180 | " | 104-5 | 102.5 |
| | " | 1,877 | " | 115 | 121 |
| | Cargo | 10,070 | " | 100 | 105 |
| | Tanker | 5,180 | " | 90-1 | 91.3 |
| | Cargo | 13,000 | " | 78.5 | 77.5 |
| | Tanker | 11,475 | " | 98.5 | 95.8 |
| | " | 12,832 | " | 80.0 | 87.5 |
| | " | 14,635 | " | 80.0 | 81.5 |
| Tobin (22) | Tanker | | 2 node vert. | 76 | 75.4 |
| | Liner | | 3 " " | 150 | 148.7 |
| | " | | 4 " " | — | 269 |
| | Tanker | 8,300 | 2 " " | 112 | — |
| Nicholls (10) | Destroyer | 1,378 | 2 node vert. | 120 | 135 |

| Author and reference | Type of Vessel | Displacement | Type of Vibration | Frequency | |
|---|---|---|---|---|---|
| | | | | Meas. | Comp. |
| Nicholls (10) | Destroyer | 1,378 | 2 node hor. | 150) to) 180) | 164 |
| | " | 1,378 | 3 node vert. | 170 | 284 |
| | " | 1,378 | 3 " hor. | 260) to ) 390) | 326 |
| | " | 1,378 | 1 node tors. | 630) to) 700) | – |
| Horn (18) | Liner | 12,750 | 2 node hor. | 125) to) 130) | 127.3 |
| Schadlofsky (6) | Tanker | 16,600 | 2 node vert. | 81 | 81.5 |
| | Cable layer | 834 | 2 " " | 205) to) 210) | 210 |
| | Destroyer | 1,378 | 2 " " | 120 | 118.2 |
| | Tanker | 8,160 | 2 " " | 112 | 112 |
| | Freighter | 8,360 | 2 " " | 105 | 104.1 |
| Lovett (23) | Passenger | | 2 node vert. | 65 | |
| | " | | 2 " " | 84 | |
| | " | | 2 " " | 67 | |
| | " | | 2 " " | 59 | |
| | Tanker | | 2 " " | 76 | |
| | " | | 2 " " | 112 | |
| Schmidt (15) | Motorship | 7,010 | 2 " " | 105) to) 108) | 106.3 |
| Taylor (24) | | 9,300 | 2 " " | 120.2 | 118 |
| | | 4,100 | 2 " " | 148 | 151 |
| | | 6,550 | 2 " . " | 100 | 98.5 |
| | | 9,050 | 2 " " | 89 | 92 |
| | | 12,700 | 2 " " | 79 | 80.5 |
| | | 6,850 | 2 " " | 92 | 93 |

| Author and reference | Type of Vessel | Displacement | Type of Vibration | Frequency | |
|---|---|---|---|---|---|
| | | | | Meas. | Comp. |
| Taylor (24) | | 9,300 | 3 node vert. | 250.5 | 236 |
| | | 8,000 | 3 " " | 214 | 206 |
| | | 12,700 | 2 " hor. | 106 | 104.5 |
| | | 6,850 | 2 " " | 147 | 142 |
| | | 8,000 | 2 " " | 137 | 135 |
| Todd (25) | Cargo | 7,940 | 2 node vert. | 100(?) | — |
| | Tanker | 15,190 | 2 " " | 78–9 | 121 |
| | " | 4,377 | 2 " " | 105–6 | — |
| | Cargo | 6,590 | 2 " " | 81–2 | 142 |
| | " | 3,835 | 2 " " | 109 | 121 |
| | Tanker | 4,180 | 2 " " | 104–5 | 124 |
| | " | 4,377 | 2 " " | 105–6 | 144 |
| | " | 1,877 | 2 " " | 115 | 198 |
| | Cargo | 10,070 | 2 " " | 100 | 125 |
| | Tanker | 5,180 | 2 " " | 90–1 | 130 |
| | Cargo | 13,000 | 2 " " | 78.5 | — |
| | Tanker | 11,475 | 2 " " | 98.5 | 121 |
| | " | 12,832 | 2 " " | 80 | 107 |
| | " | 14,635 | 2 " " | 80 | 93 |
| | " | 6,400 | 2 " " | 112 | 138 |
| | " | 8,200 | 2 " " | 112 | 124 |
| | " | 15,700 | 2 " " | 72–76 | — |
| Roop (20) | Oiler | 15,430 | 2 node vert. | 60.3 | 63.3 |
| | " | 7,600 | 2 " " | 88.5 | — |
| | " | 12,600 | 2 " " | 81.1 | — |
| | " | 10,000 | 2 " " | 73.5 | — |
| | Battleship | 32,000 | 2 " " | 82 | — |
| Cole (4) | Tanker | 5,800 | 2 " " | 112 | 112 |

In the case of those vessels for which sufficient data were available the equivalent values of the Schlick constant required to check the measured frequencies were calculated. These values are tabulated in Table VI.

Table VI – Experimental Values of Schlick Constants

| Author and Reference | Type of Vessel | Displacement (tons) | Overall Length (feet) | Moment of Inertia amidships (ft$^2$in$^2$) | $\sqrt{\dfrac{I}{DL^3}}$ x 10$^4$ | Measured Frequency (per min.) | Equiv. Schlick Constant x 10$^{-5}$ |
|---|---|---|---|---|---|---|---|
| Todd (8) | Tanker | 15,190 | 440 | 476,000 | 6.08 | 78.9 | 1.30 |
| Tobin (22) | " | 8,300 | 350 | 233,890 | 8.11 | 112 | 1.38 |
| Nicholls (10) | Destroyer | 1,378 | 310 | 33,000 | 8.96 | 120 | 1.34 |
| Schmidt (15) | Motorship | 7,010 | 484 | 718,000 | 9.50 | 106 | 1.12 |
| Schadlofsky (6) | Tanker | 16,600 | 462 | 604,000 | 6.08 | 81 | 1.33 |
| | Cable layer | 834 | 181 | 15,600 | 17.8 | 207.5 | 1.17 |
| | Tanker | 8,160 | 366 | 234,000 | 7.65 | 112 | 1.47 |
| | Freighter | 8,360 | 371 | 264,000 | 7.88 | 105 | 1.33 |
| Cole (4) | Tanker | 8,151 | 350 | 233,890 | 8.19 | 112 | 1.37 |
| E.M.B. Report #372 | Destroyer | 1,382 | 310 | 35,000 | 9.22 | 107 | 1.16 |
| Roop (20) | Tanker | 15,430 | 475 | 447,000 | 5.20 | 60.3 | 1.16 |
| | Battleship | 32,000 | 583 | 1,325,000 | 4.57 | 77.1 | 1.69 |

The average of the Schlick constants tabulated in Table VI is 1.32 x 10$^5$, and the average deviation from this value is ±8.6 per cent.

### 7. Conclusions

The average deviation in the experimentally determined value of Schlick's constant for the ships tabulated in the previous section compares very favorably with the estimated probable error of ±5 per cent for the theoretical computations. Hence because of its simplicity the Schlick formula is to be preferred in the major-ity of cases. Sufficient data are not now available for selecting a value of Schlick's constant according to the class of vessel, and hence the average value 1.32 x 10$^5$ is at present the best for all types. With the accumulation of data in the near future it should be possible to select a better value according to the type of ship.

The graphical methods of Taylor and Schadlofsky and the tabular method of Lewis are accurate for computing the theoretical frequency but the subsequent cor-rections to the theoretical frequency remain in some doubt. Of these corrections only those for shear deflection and water effect are of sufficient magnitude to be taken into account. The shear deflection lowers the frequency by about 5 per cent and the water effect lowers it by about 25 per cent.

The flexure theory is not applicable to the computation of higher harmonics and the only guide here is the collection of data on other ships.

From the measurements tabulated in section 6 it appears that the two noded vertical frequencies fall within a relatively narrow range on either side of 100 vibrations per minute, and hence in many cases the frequency could be estimated directly from this table. The close relation between the two noded vertical frequency and the strength problem, however, lends importance to the theoretical calculation. If the water effect were definitely known the effective EI, which is the measure of bending strength, could readily be deduced from the natural frequency. This is shown by the close agreement between measured and computed frequencies for the models when vibrated out of water.

## Acknowledgment

## 8. Bibliography

| | | |
|---|---|---|
| 1. | O. Schlick: | Articles in T.I.N.A., 1884, 1893, 1894, 1901, 1911. |
| 2. | Lord Rayleigh: | "Theory of Sound", 2 Ed. Vol. 1, pp. 111 and 287, 1877-8. |
| 3. | J. Morrow: | "On the Lateral Vibration of Bars of Uniform and Varying Sectional Area", Phil. Mag., Vol. 10, 1905. |
| 4. | A. P. Cole: | "The Natural Periods of Vibrations of Ships", Institute of Engineers and Shipbuilders, Scotland, Vol. LXXII, 1928-9, pp. 43-86. |
| 5. | J. L. Taylor: | "Ship Vibration Periods", N.E.C.I., 1927-8, pp. 143-176. |
| 6. | E. Schadlofsky: | "Über Rechung und Messung der Elastischen Eigen-schwingungen von Schiffskorpern", S.T.G., Nov. 1932. (This paper is translated in E.M.B. Report No. 382). |
| 7. | F. M. Lewis: | "Vibration and Engine Balance in Diesel Ships", N.A.M.E., 1927, pp. 259-286. |
| 8. | F. H. Todd: | "Ship Vibration — A comparison of Measured with Calculated Frequencies", N.E.C.I., 1933. |
| 9. | J. B. Henderson: | "The Vibration of Ships and the Use of a Dynamic Model for Determining the Elasticity of Ships", T.I.N.A., 1909, pp. 29-46. |
| 10. | H. W. Nicholls: | "Vibration of Ships", T.I.N.A., 1924, pp. 141-163. |

11. E. B. Moullin
    and A. D. Brown:     "On the Periods of a Free-free Bar Immersed in Water",
                         Cambridge Phil. Soc. Proc., Vol. XXIV, pp. 400-413,
                         May 1928.

12. F. M. Lewis:        "The Inertia of Water Surrounding a Vibrating Ship",
                         N.A.M.E., Nov. 14, 1929, pp. 1-20.

13. J. J. Koch:         "Eine Experimentelle Methode zur Bestimmung der
                         Reduzierten Masse des Mitschwingenden Wassers bei
                         Schiffschwingungen", Ingenieur-Archiv, IV Band, 2
                         Heft, 1933, pp. 103-109.

14. A. Dimpker:         "Über Schwingende Korper an der Oberflache des
                         Wassers", W.R.H., 15 Jan., 1934, pp. 15-19.

15. F. Schmidt:         "Die Berechung von Schiffschwingungen", W.R.H.,
                         22 March 1930, Heft 6, pp. 124-128.

16. L. Gumbel:          "Torsional Vibrations of Elastic Shafts of Any Cross
                         Section and Mass Distribution and Their Application to
                         the Vibration of Ships", T.I.N.A. 1912, Vol. LIV,
                         p. 219.

17. F. M. Lewis:        "Torsional Vibration of Irregular Shafts", Journal
                         Am. Soc. Naval Engrs., Nov. 1919, p. 857.

18. F. Horn:            "Horizontal und Torsions - Schiffschwingungen auf
                         Frachtschiffen", W.R.H., Vol. 6, 1925, Heft 18,
                         pp. 541-585.

19. G. Vedeler:         "Torsion of Ships", T.I.N.A., 1922.

20. W. P. Roop:         "Natural Frequencies of Hull Vibration", N.A.M.E.,
                         Nov., 1930, pp. 135-145.

21. R. Bernhard:        "Rein Dynamische Verfahren zur Unterschung Der
                         Beanspruchungen von Bauwerken", Der Stahlbau, 1929,
                         Heft 6.

22. T. C. Tobin:        "A Method of Determining the Natural Periods of
                         Vibration of Ships", T.I.N.A., 1922, pp. 153-165.

23. W. L. Lovett:       "The Vibration of Ships", Shipbuilding and Shipping
                         Record, Jan., 1927, pp. 11-12.

24. J. L. Taylor:       "Vibration of Ships", T.I.N.A., 1930, pp. 162-173.

25. F. H. Todd:         "Some Measurements of Ship Vibration", N.E.C.I.,
                         Nov. 1931.

26. C. E. Inglis:       "Suggested Method of Minimizing Vibrations in Ships",
                         T.I.N.A., April 7, 1933.

27. G. E. Pavlenko:     "A Method of Calculating Ship Vibration Frequencies",
                         Engineering, 1926, June 25, pp. 748-9.

28. E. M. B. Report
    No. 372:          "Vibration Tests on U.S.S. HAMILTON".
29. Wm. Hovgaard:     "Structural Design of Warships".